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PRECISION CALCULATIONS FOR FUTURE COLLIDERS

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I discuss the motivations for, and the status of, precision calculations for the Large Hadron Collider (LHC) and the planned International Linear Collider (ILC).

1. Why do we care?

In less than a year, the CERN Large Hadron Collider (LHC) will begin operation. The LHC will collide protons at a center-of-mass energy of $\sqrt{s} = 14$ TeV with a design luminosity of $\mathcal{L} = 10^{34}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. This represents an increase of a factor of seven in energy and a factor of 100 in luminosity over the Fermilab Tevatron. With its unprecedented energy and luminosity, the LHC promises to revolutionize particle physics. It will unveil the mechanism of electroweak symmetry breaking (EWSB) and shed light on the physical processes that are responsible for the origin of mass. The LHC holds the potential to make dark matter in the laboratory and perhaps even to reveal extra dimensions of space. Its reach for uncovering new phenomena is dramatically higher than that of all previous accelerators. The LHC truly will be a discovery machine.

For the next decade, the particle physics community is planning to build a linear e^+e^- collider with a center of mass energy in the range of $500 - 1000$ GeV and a luminosity of $\mathcal{L} = 2 \times 10^{34}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. An e^+e^- collider will provide a cleaner environment than a hadron collider and will complement the LHC in its search for new physics[1].

To uncover the mechanism of EWSB and discover new physics at the LHC, it is necessary to have accurate theoretical calculations of Standard Model (SM) processes and new physics signatures. The final states of many processes are quite complex at the LHC. The lowest-order (LO) predictions for many SM processes exhibit a significant dependence on the unphysical renormalization and factorization scales that can be traced to the truncation of the perturbation series. The scale dependence can be reduced by calculating observables to higher order in perturbation theory. Higher-order QCD and, in some cases, electroweak (EW) radiative corrections are needed for accurate SM predictions. Sometimes, such as for W and

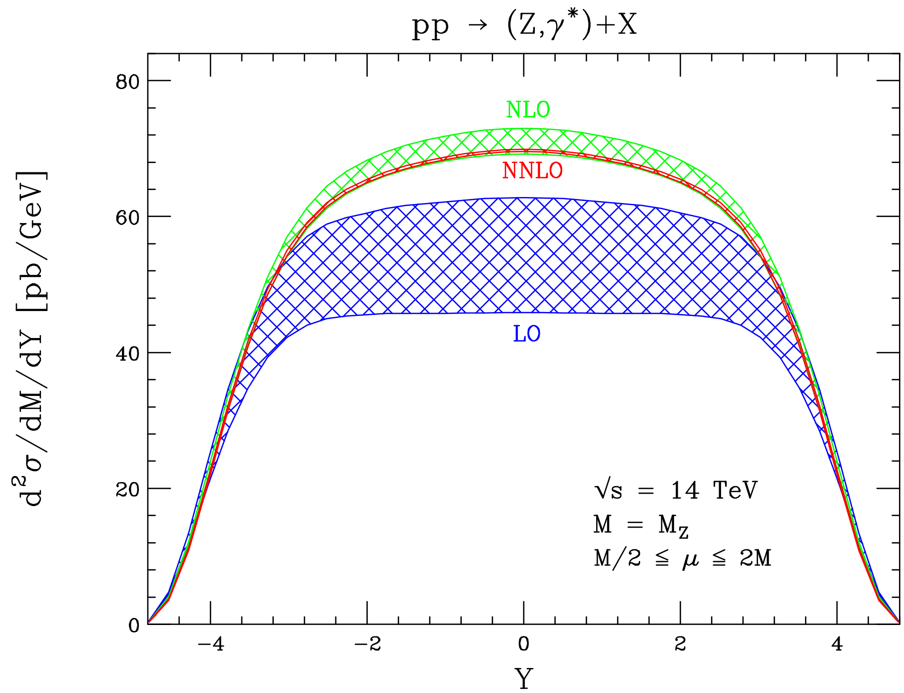

Fig. 1. The Z boson rapidity distribution at the LHC. Shown are the LO, NLO and NNLO predictions (from Ref. [2]).

Z production[2], the reduction of the scale dependence once higher order QCD corrections are taken into account is dramatic. This is illustrated in Fig. 1 where the Z boson rapidity distribution at the LHC is shown. While the LO cross section varies by about 50% for a renormalization/factorization scale μ in the range $M_Z/2 \leq \mu \leq 2M_Z$, the uncertainty at next-to-leading order (NLO) is reduced to about 10%, and at next-to-next-to-leading order (NNLO) to about 1%.

Although much has been accomplished in recent years, much remains to be done in order to ensure that the full physics potential of the LHC can be utilized. Recent results relevant for the LHC are discussed in Sec. 3.1. Here, without going into any details, I give a time ordered "LHC shopping list" of precision calculations which are still needed:

(1) For $10 - 30$ fb^{-1} (2009 – 2010):
  (a) compute full NLO QCD corrections to $pp \to t\bar{t} \to b\bar{b} + 4f$
  (b) compute full tree level calculation of $t\bar{t}Wjj$ production
  (c) compute NLO QCD corrections to $t\bar{t}j$, $t\bar{t}\gamma$, $t\bar{t}b\bar{b}$, $t\bar{t}jj$ and $WWjj$ production
  (d) resum QCD corrections to $qq' \to qq'H$

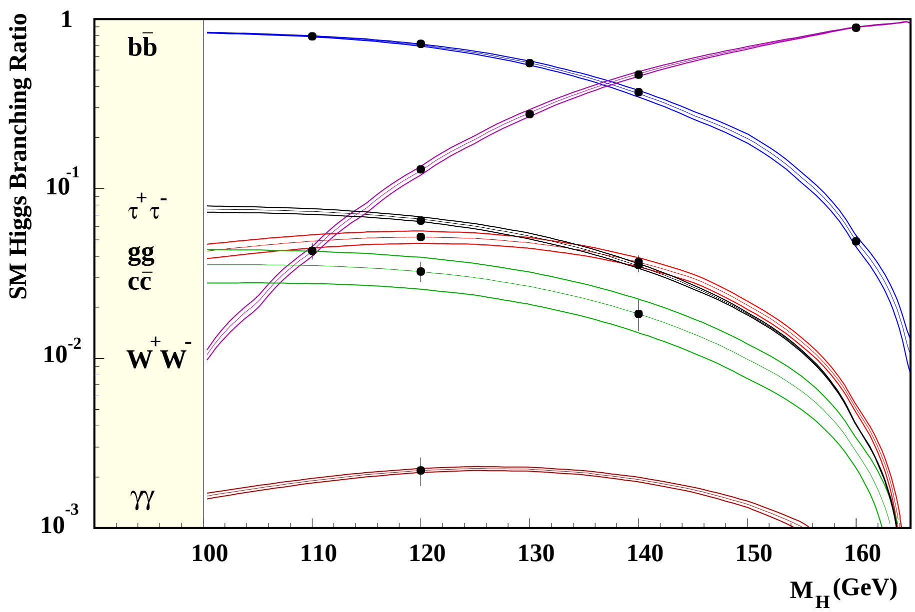

Fig. 2. The predicted SM Higgs boson branching ratios. Points with error bars show the expected experimental accuracy, while the lines show the estimated uncertainties on the SM predictions. (from Ref. [1]).

(2) For 300 fb^{-1} (2012 – 2013): compute NLO QCD corrections to $gg \to HH$, $t\bar{t}W$ and $t\bar{t}Z$ production

(3) For 3000 fb^{-1} (SuperLHC, $>$ 2015): compute NLO QCD corrections to $WWWjj$, $jj\gamma\gamma$ and $Q\bar{Q}\gamma j$ production

The enormous center of mass energy of the LHC makes it an ideal tool to search for new particles which are a common prediction of all new physics scenarios. On the other hand, the cleaner environment of the ILC will make it easier to precisely measure SM observables, such as the W mass. Their measurement is expected to yield complementary information on new physics. New heavy particles, which are a common prediction of all beyond-the-SM models, generally contribute to observables via virtual radiative corrections, and thus lead to small deviations from the SM predictions.

Of particular interest at the ILC is the precise measurement of the Higgs boson couplings to fermions and the weak bosons (once a Higgs candidate particle has been found). At the LHC, these couplings can be measured with a precision of $\mathcal{O}(10\%)$ at best[3]. At the ILC it will be possible to determine the Higgs boson couplings with an accuracy of a few percent[1]. Figure 2 shows the predicted branching ratios for various Higgs decays, the expected experimental precision, and the current theoretical uncertainties as a function of the Higgs boson mass. If the Higgs boson mass

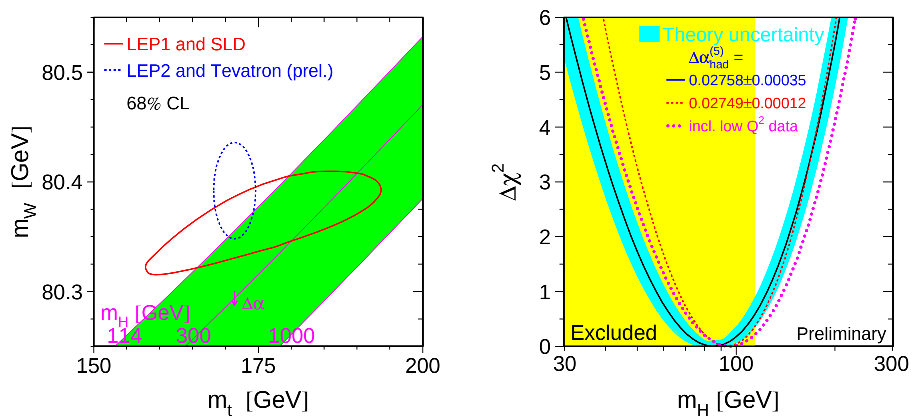

Fig. 3. Left pane: the experimentally allowed regions in the $m_W - m_t$ plane are shown together with the SM predictions. Right pane: $\Delta\chi^2 - \chi^2 - \chi^2_{min}$ vs. m_H curve. The line is result of a fit to all electroweak data, and the blue band represents an estimate of the theoretical error due to missing higher order corrections. The vertical yellow band represents the 95% CL bound from searches at LEP2[7].

is ≤ 140 GeV, it may also be possible to measure the Higgs boson self-coupling, λ_{HHH}, at the ILC, and thus to directly probe the Higgs potential[4]. At the LHC, λ_{HHH} can be probed in this mass range[5] only once the luminosity has been been upgraded to $\mathcal{L} = 10^{35}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. In order to probe the Higgs boson couplings at the ILC with the advertised accuracy, the one-loop electroweak radiative corrections to $e^+e^- \to ZH$, $e^+e^- \to \nu\bar{\nu}H$, $e^+e^- \to e^+e^-H$, $e^+e^- \to t\bar{t}H$, $e^+e^- \to ZHH$, $e^+e^- \to \nu\bar{\nu}HH$, and $e^+e^- \to e^+e^-HH$ are needed. Thanks to new automated tools which I will discuss in more detail in the following Section, the one-loop electroweak radiative corrections to all these processes except $e^+e^- \to e^+e^-HH$ have been calculated in the last few years[6].

Other electroweak observables which can be precisely measured at the ILC are the W mass, m_W, and the effective weak mixing angle, $\sin^2\theta_{eff}$. The one-loop electroweak corrections to these parameters depend quadratically on the top quark mass, m_t, and logarithmically on the Higgs boson mass, m_H. Thus, measuring m_W or $\sin^2\theta_{eff}$ and m_t makes it possible to extract information on m_H. The regions currently allowed by LEP1 and SLC data, and LEP2 and Tevatron data in the $m_W - m_t$ plane, together with the SM prediction are shown in Fig. 3a. Figure 3b shows the $\Delta\chi^2$ curve as a function of m_H. The blue band represents an estimate of the theoretical error due to missing higher order corrections. It is dominated by the theoretical uncertainty on the effective weak mixing angle for which the full two-loop electroweak corrections are now known[8]. Still, the remaining theoretical

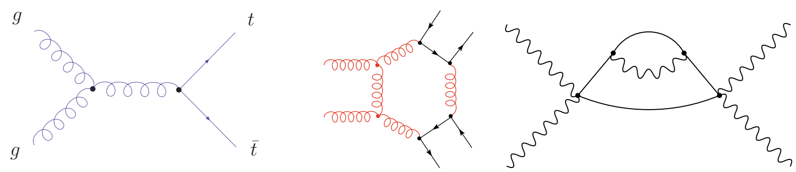

Fig. 4. Examples for tree level (left), one-loop (middle) and two-loop (right) Feynman diagrams.

uncertainty is

$$\delta \sin^2 \theta_{eff}^{theor} \approx (4-5) \times 10^{-5}, \tag{1}$$

which is only a factor 3−4 smaller than the experimental uncertainty of $\delta \sin^2 \theta_{eff}^{exp} = 0.00017$[9]. I will come back to the measurement of $\sin^2 \theta_{eff}$ and the W mass at the ILC in Sec. 3.2.

The reminder of this review is organized as follows. In Sec. 2, I briefly outline the framework of higher order calculations in perturbation theory and briefly discuss the tools which currently available for performing these calculations. In Sec. 3, I discuss recent results which are relevant for the LHC and ILC. Section 4 contains a summary and outlook.

2. Tools for Loop Calculations

The theoretical framework for loop calculations in high energy physics is perturbation theory. In perturbation theory, the observable of interest is expanded in powers of a (small) coupling constant. The lowest order (LO) terms correspond to tree level Feynman diagrams, the NLO corrections involve one-loop diagrams, and so on. Examples for tree level, one-loop and two-loop diagrams are shown in Fig. 4.

The general strategy of a loop calculation is best illustrated using a simple process such as 2 jet production in e^+e^- collisions, $e^+e^- \to q\bar{q}$, as an example. In order to compute the NLO QCD corrections to $e^+e^- \to q\bar{q}$, one needs to calculate the one-loop corrections to this process, and the tree level process $e^+e^- \to q\bar{q}g$. Both occur at the same order in perturbation theory. Due to soft and collinear divergencies, the cross section for $e^+e^- \to q\bar{q}$ at one-loop and for $e^+e^- \to q\bar{q}g$ each diverges; however, their sum is finite and represents the physical cross section for 2 jet production in e^+e^- collisions at NLO in QCD.

Computing the $e^+e^- \to q\bar{q}g$ cross section is the easy part of the calculation. Tree level calculations are technically straightforward, and a number of automatic programs exists which greatly simplify the task. The most general and flexible tools are `MadEvent`[10], `Grace`[11], `CalcHEP`[12], `CompHEP`[13], and `WHIZARD`[14] which allow the user to completely specify the process he/she wishes to calculate. The flexibility of these programs comes at the price of speed. `ALPGEN`[15], on the other hand, is extremely fast, but works only for a selected set of processes which are hard coded into the program. `MadEvent`, however, can be run in parallel on several machines,

Table 1. Outline of a loop calculation.

draw all possible diagrams	topological task
which particles run in given diagram	combinatorial task
translate diagrams into formulas via Feynman rules	database look-up
contract Lorentz indices; take traces	algebraic manipulation
reduce to known/master integrals	algebraic manipulation
cancel IR and/or UV singularities	algebraic manipulation
translate output into computer program	programming
run program	wait, drink coffee

and a future version of `CompHEP` may be able to do the same[16]. This (partially) compensates the relative slowness of these programs.

In addition to calculating the cross sections for SM processes, most programs are now able to also compute beyond-the-SM (eg. supersymmetric) processes. The capabilities of programs which automatically calculate tree level processes is only limited by the computing power available to the user. For multi-particle final states, thousands of Feynman diagrams may contribute; the number of diagrams grows factorially with the number of final state particles. For the process $e^+e^- \to W^+W^-\bar{b}bjj$, for example, there are 4896 Feynman diagrams. The numerical evaluation of matrix elements takes progressively more time the more Feynman diagrams contribute.

The calculation of the one-loop corrections to $e^+e^- \to q\bar{q}$ is considerably more involved. The general strategy of a loop calculation is outlined in Table 1. Even for moderately complicated processes such as $e^+e^- \to 4$ fermions the number of Feynman diagrams which has to be calculated can be extremely large ($\mathcal{O}(10^4)$). Additional complications arise from large cancellations which occur between certain Feynman diagrams. This requires extra care with the numerical implementation.

Because of the complexity of loop calculations, automatic tools are essential to accomplish the goal. So far, program packages for automated loop calculations only exist for electroweak one-loop corrections. The `Grace/1-loop`[17] package has been used successfully in calculating a number of processes relevant for the ILC. Another popular set of semi-automatic tools are `Feynarts`[18], `FeynCalc`[19], `FormCalc`[20] and `LoopTools`[20]. Finally, `Diana`[21] is an automatic tool for generating the Feynman diagrams which contribute to a given process and a given order in perturbation theory. `Diana` also produces the input needed for programs which evaluate the traces of Dirac-matrices, such as `FORM`[22]. Not surprisingly, the structure of these programs is fairly complex. As an example, I show the flow diagram of the `Grace/1-loop` package in Fig. 5. Packages for automated calculations of one-loop QCD corrections are currently under development (`Grace-QCD`[24] and `Samper`[25]).

The complexity of loop calculations rapidly increases with the number of loops, and with the number of particles in the final state. On the other hand, the cross section of processes falls rather quickly with the number of final state particles.

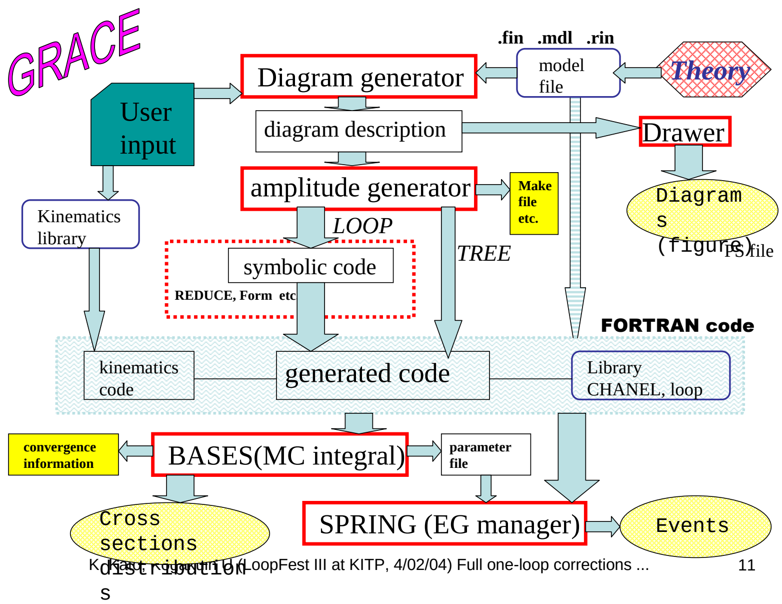

Fig. 5. Flow diagram of the `Grace/1-loop` package. (from Ref. [23]).

Thus, the requirements on the theoretical accuracy for $2 \to n$, $n > 2$, processes are less than for $2 \to (n-1)$ processes, ie. the order in perturbation theory up to which one needs to calculate cross sections decreases with increasing number of particles in the final state. Figure 6 shows the loop order (which is equivalent to the order of perturbation theory) which one needs to calculate for processes of interest at the ILC as a function of the number of particles in the final state. The figure also shows the current status of calculations of higher order corrections for these processes.

3. Recent Results

In this Section I discuss some recent results in precision calculations relevant for LHC and ILC processes.

3.1. *One-loop Corrections for LHC Processes*

Multijet production is an important background for many processes of interest at the LHC. The NLO QCD corrections to 2 jet and 3 jet production have been known for several years[27,28]. In contrast, the calculation of the NLO QCD corrections to 4 jet production is just beginning. The most complicated contribution to the one-loop corrections to $pp \to 4$ jet originates from $gg \to gggg$. The one-loop corrections to this sub-process were recently calculated in Ref. [29]. Approximately

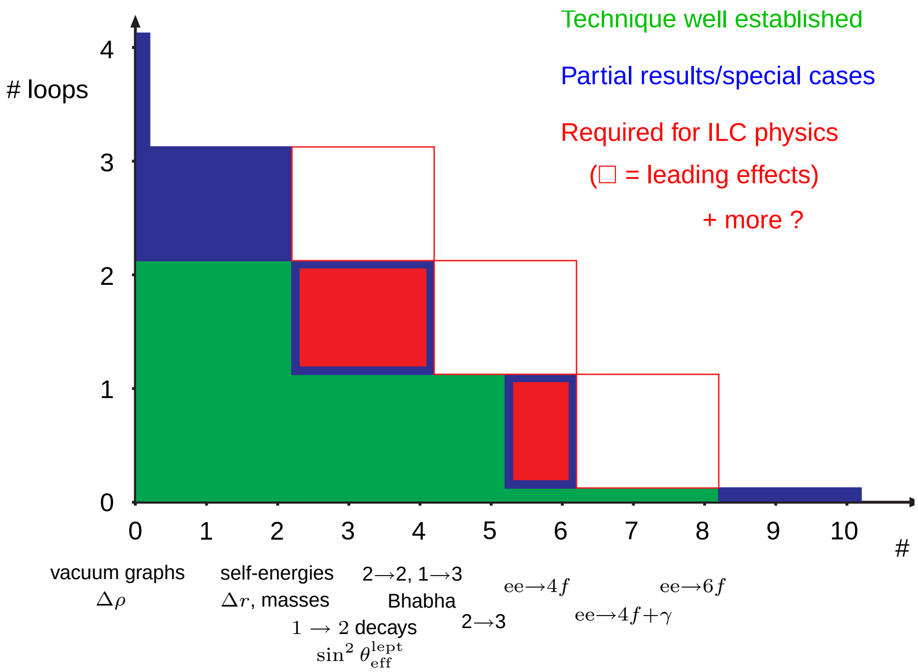

Fig. 6. The present state of art of calculations of higher order corrections for ILC processes (from Ref. [26]).

10,000 Feynman diagrams contribute to $gg \to gggg$ at the one-loop level. To obtain numerical results, a new promising method was used which semi-numerically evaluates loop integrals[30].

In Sec. 1, I noted that calculating higher order QCD corrections usually reduces the sensitivity of the cross section on the renormalization and factorization scales, μ_R and μ_F. A recent calculation[31] of the NLO QCD to W^+W^- production via vector boson fusion (VBF), $qq' \to W^+W^-qq'$, nicely illustrates this point. W pair production via VBF is one of the most important Higgs discovery channels at the LHC[32]. While the LO $qq' \to W^+W^-qq'$ cross section varies very strongly with μ_R and μ_F, the NLO cross section is almost independent of the choice of scale over a wide range (see Fig. 7).

As mentioned in Sec. 2, one has to include real quark/gluon radiation diagrams when calculating higher order corrections in QCD in order to obtain a finite cross section. Individually, the cross sections obtained from virtual and real corrections are infinite due to soft and collinear divergencies. This also happens when calculating QED radiative corrections. The soft and collinear divergencies are due to the vanishing mass of the QCD and QED gauge bosons.

In contrast to QCD and QED, the electroweak gauge bosons are massive and act as infrared regulators. Cross sections for real and virtual weak corrections thus are separately finite. The virtual weak corrections turn out to become large and

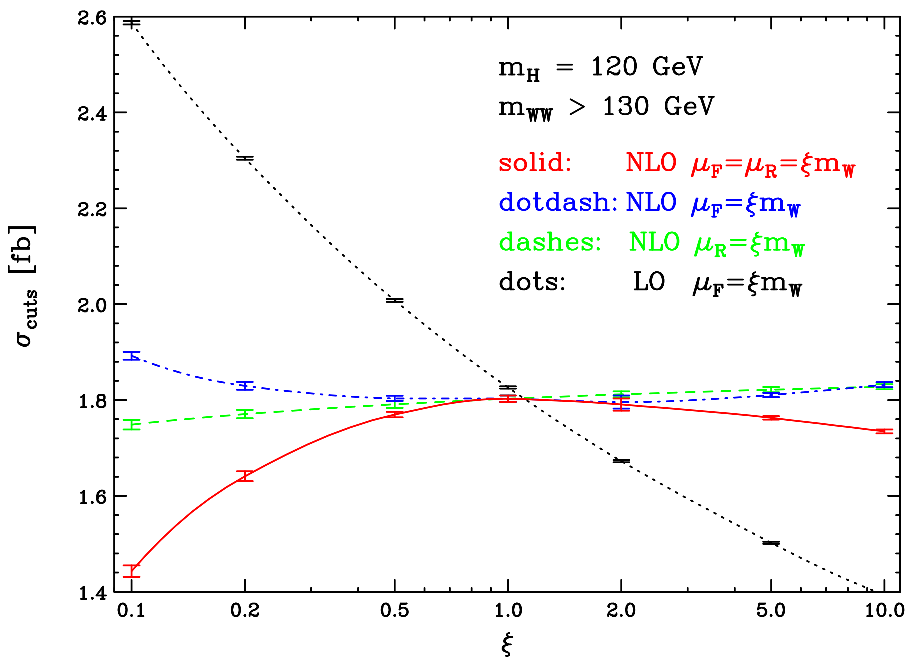

Fig. 7. The scale dependence of the $qq' \to W^+W^-qq'$ cross section for pp collisions with $\sqrt{s} =$ 14 TeV at LO and NLO (from Ref. [31]).

negative at high energies, due to the presence of Sudakov-like logarithms of the form $(\alpha/\pi)\log^2(\hat{s}/m^2_{W,Z})$, where $\hat{s}$ is the squared parton center of mass energy, and $m_{W,Z}$ is the mass of the W or Z boson. For $\sqrt{\hat{s}} \geq 1$ TeV, the $\mathcal{O}(\alpha)$ one-loop EW radiative corrections can easily become larger in magnitude than the $\mathcal{O}(\alpha_s)$ QCD corrections.

The Sudakov-like logarithms originate from collinear and infrared divergences which would be present in the limit of vanishing W and Z masses and are well understood[33,34,35,36,37,38]. The appearance of large logarithms in one-loop weak corrections has recently been demonstrated in a number of explicit calculations. For hadron colliders, the $\mathcal{O}(\alpha)$ virtual weak corrections to inclusive jet[39], isolated photon[40,41], $Z + 1$ jet[41,42], Drell-Yan[43,44,45,46,47,48], di-boson[49,50,51], $\bar{t}t$[52,53,54,55], and single top production [56,57,58] have been calculated. As an example, I show the virtual weak corrections for the photon transverse momentum distribution in $pp \to W\gamma \to e\nu\gamma$ in Fig. 8. They strongly increase in magnitude with increasing photon transverse momentum and reach about -25% at $p_T(\gamma) = 800$ GeV. In almost all of the calculations where large logarithms appear in one-loop weak corrections, weak boson emission diagrams have not been taken into account, although they contribute at the same order in perturbation theory as the one-loop corrections and often contribute substantially to the NLO electroweak cross section[59].

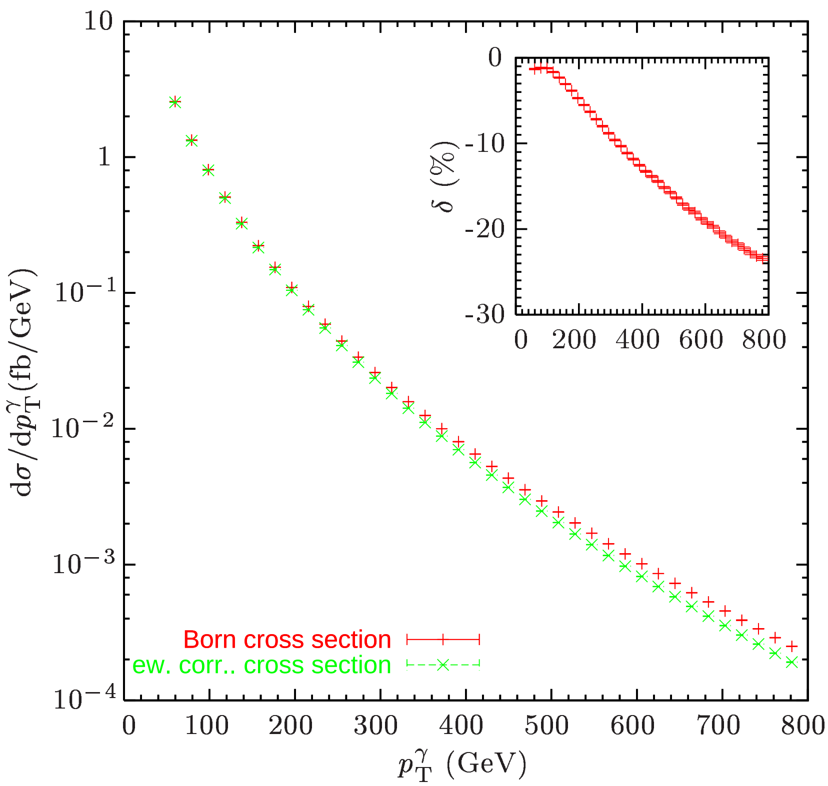

Fig. 8. The photon transverse momentum distribution in $pp \to W\gamma \to e\nu\gamma$ for $\sqrt{s} = 14$ TeV at LO and including weak virtual corrections (from Ref. [50]). The inset shows the relative corrections.

3.2. *Recent Calculations relevant for the ILC*

As mentioned in Sec. 1, it will be possible to measure the effective weak mixing angle and the W mass at the ILC. The ILC detectors can be calibrated by measuring the Z boson mass and comparing the result to the value obtained at LEP1. This requires operation of the ILC at the Z peak, which also offers a chance to determine $\sin^2\theta_{eff}$. The W mass can either be measured by directly reconstructing the W bosons in $e^+e^- \to W^+W^-$, or by measuring the the W pair cross sections in the threshold region ($\sqrt{s} \approx 161$ GeV). The latter method promises to be more precise at the ILC.

As noted before, the full two loop electroweak corrections to the effective weak mixing angle were recently calculated[8]. This, however, will not be sufficient for the ILC. At the ILC one hopes to measure $\sin^2\theta_{eff}$ with a precision of [60]

$$\delta \sin^2\theta_{eff}^{exp} = 1.3 \times 10^{-5}, \tag{2}$$

which about a factor three smaller than the current theoretical uncertainty from unknown higher order corrections (see Eq. (1)). For a measurement of the effective weak mixing at the ILC, one thus has to calculate the 3-loop $\mathcal{O}(\alpha_s\alpha^2)$ corrections.

The W mass can be measured with a precision of about 7 MeV in a threshold scan at the ILC[61]. This means that the $e^+e^- \to 4$ fermion cross section has to be known with a precision of [62]

$$\frac{\Delta\sigma}{\sigma} \approx 5 \times 10^{-4} \tag{3}$$

in the threshold region. The uncertainty of the LO $e^+e^- \to 4$ fermion cross section at $\sqrt{s} = 161$ GeV is approximately[63]

$$\left(\frac{\Delta\sigma}{\sigma}\right)_{LO} = 0.014. \tag{4}$$

For a W mass measurement from a threshold scan at the ILC, one therefore needs the full $\mathcal{O}(\alpha)$ electroweak radiative corrections to $e^+e^- \to 4$ fermions. These corrections have recently been calculated[64]. A major complication which had to be overcome in this calculation is how to include finite W width effects while maintaining gauge invariance. Including the W width in the W propagator corresponds to a resummation of the imaginary part of the W vacuum polarization, and is essential in the threshold region for obtaining a realistic prediction of the cross section. Since only a subset of the Feynman diagrams which contribute to $e^+e^- \to 4$ fermions is resummed in this procedure, this will break gauge invariance. The gauge invariance problem was solved in Ref. [64] by using the complex mass scheme and complex renormalization.

A main technical challenge in the calculation of the full $\mathcal{O}(\alpha)$ electroweak radiative corrections to $e^+e^- \to 4$ fermions is the reduction of hexagon diagrams to box diagrams which, employing conventional methods, leads to numerical instabilities. Ref. [64] overcame this problem by using Cayley determinants[65]. To illustrate the results obtained in Ref. [64], I show the relative corrections to the $e^+e^- \to \tau^+\nu_\tau\mu^-\bar{\nu}_\mu$ cross section in Fig. 9. In the W threshold region, the difference between the full and approximate $\mathcal{O}(\alpha)$ electroweak radiative corrections is seen to be approximately 2%.

The calculation of the full $\mathcal{O}(\alpha)$ electroweak radiative corrections to $e^+e^- \to 4$ fermions represents a major step forward. Many other calculations of radiative corrections to $2 \to 4$ processes are now feasible. However, the $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to 4$ fermions may not be sufficient for a W mass measurement with a precision of 7 MeV at the ILC. Next-to-leading logarithmic electromagnetic corrections of order $(\alpha/\pi)^2\log(m_e^2/s)$, where m_e is the electron mass, and higher order effects associated with the Coulomb singularity may modify the $e^+e^- \to 4$ fermion cross section by $\mathcal{O}(10^{-3})$[64,66,67]. These corrections still have to be calculated.

3.3. *Recent Results from the Two-loop Frontier*

Enormous progress in calculating two-loop corrections has been made in the last few years. Key developments have been the algebraic reduction of loop integrals to

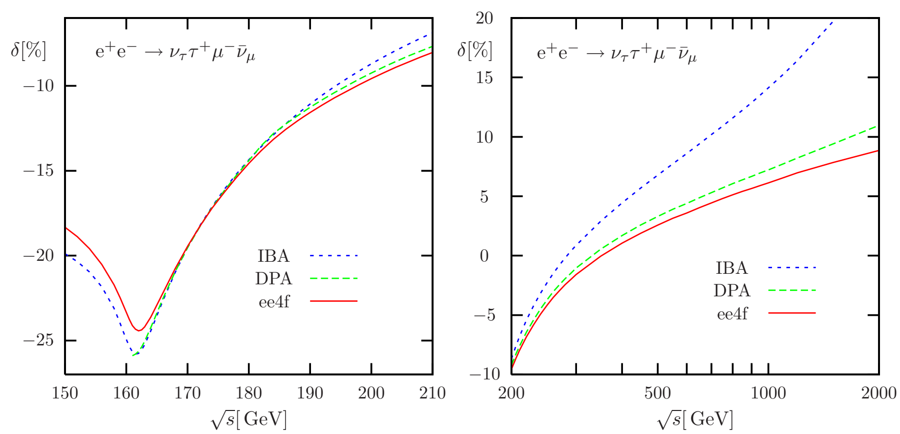

Fig. 9. The relative $\mathcal{O}(\alpha)$ electroweak radiative corrections to $e^+e^- \to \tau^+\nu_\tau\mu^-\bar{\nu}_\mu$ as a function of the center of mass energy, $\sqrt{s}$. Shown are the results for the improved Born approximation (IBA), the $\mathcal{O}(\alpha)$ electroweak radiative corrections in the double pole approximation (DPA), and the full $\mathcal{O}(\alpha)$ electroweak radiative corrections (ee4f) (from Ref. [64]).

master integrals by integration by parts and Lorentz invariance identities, and the calculation of master integrals using the Mellin-Barnes technique, differential equations and numerical techniques[68]. This has lead to a number of results for explicit two-loop amplitudes such as 2 jet production at the LHC[69], $e^+e^- \to 3$ jets[70], and $e^+e^- \to e^+e^-$[71]. Of course, for a full calculation of the NNLO corrections to these processes, the two-loop amplitudes have to be combined with the relevant one-loop $2 \to 3$ and $2 \to 4$ amplitudes, and the tree level $2 \to 4$ and $2 \to 5$ amplitudes. This requires the development of a suitable subtraction method for the soft and collinear divergencies which appear in the calculation. Several promising techniques[72,73,74] are currently pursued.

For some processes, such as $H \to \gamma\gamma$[75], $W \to \ell\nu$ and $Z \to \ell^+\ell^-$[2] production in hadronic collisions, the fully differential NNLO QCD cross section is already available. This makes it possible to study in detail how QCD corrections affect the experimental acceptances for these processes. The W (and Z) boson cross section can be used as a luminosity monitor at the LHC[76]. This requires the theoretical uncertainty on the cross section to be below 1%. Knowledge of the NNLO QCD corrections to the fully differential W cross section is an essential ingredient to achieve this goal. $H \to \gamma\gamma$ is an important Higgs discovery channel if $m_H < 140$ GeV[77].

3.4. *New Theoretical Developments*

Recent progress in the analytical computation of tree-level[78] and massless one-loop[79] gauge theory amplitudes provides a promising alternative to the techniques used so far. This work, including new methods based on twistor-space string theories[80], has led to compact expressions and recursion relations that promise a faster numerical evaluation of differential cross sections. The next steps in bringing this approach to fruition are to generalize the results for massless one-loop diagrams to the massive case, and to build parton-level MC programs for processes of interest.

3.5. *Recent Results for Supersymmetric Theories*

In addition to precision calculations in the framework of the SM, many such calculations have been performed for supersymmetric theories. It is impossible to mention all results here so I concentrate on a few selected calculations.

The one-loop radiative corrections to the W mass in the minimal supersymmetric SM (MSSM) have been known for more than 10 years[81,82,83,84,85]. More recently, the Yukawa corrections to the ρ-parameter[86] and the Higgs masses and widths in MSSM[87] have been computed. For a recent review of electroweak precision observables in the MSSM see Ref. [88]. The one-loop corrections to chargino and neutralino pair production in e^+e^- collisions have been evaluated in Refs. [89] and [90]. New tools for supersymmetric processes include `Prospino 2.0` which computes next-to-leading order cross sections for the production of supersymmetric particles at hadron colliders[91], `Sdecay` which calculates the decay widths and branching ratios of all supersymmetric particles in the MSSM[92], and `SUSY-Madgraph` which generates complete tree level matrix elements for the production of supersymmetric particles, including decays and spin correlations[93].

4. Summary and Outlook

Accurate theoretical predictions for SM and beyond-the-SM processes are needed in order to correctly interpret data from the LHC and ILC. In the last few years enormous progress has been made in developing new techniques for loop calculations and new tools for tree level calculations for complex final states. The one-loop corrections for essentially all $2 \to 2$ processes of interest are known. Thanks to automated tools, calculations of one-loop corrections to $2 \to 3$ processes have become fairly routine. The frontier in one-loop corrections now are $2 \to 4$ processes, such as $e^+e^- \to 4$ fermions.

Although much has been accomplished, there remain significant challenges. For example, the one loop corrections for $2 \to 5$ processes such as $pp \to WWWjj$, which is a background relevant for the determination of the Higgs boson self-coupling in Higgs pair production at the LHC[94], have not been tackled yet. While the two loop corrections for many $2 \to 2$ processes are known, these have yet to be combined with the one-loop amplitudes of the relevant $2 \to 3$, and the tree level amplitudes of the corresponding $2 \to 4$ processes in order to obtain physical cross sections. The fully differential cross section including NNLO corrections is only known for a few processes. Beyond the two-loop level, calculations and tools are still in their infancy.

Approximately 85% of the work done on precision calculations is carried out in Europe or Asia, as evidenced by the references included in this review. Clearly, a stronger role of the Americas in this field which is of vital importance for the LHC and ILC, and thus more regional balance, is desirable. I hope that with the LHC approaching this will happen.

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